Lesson **1**

Set of Rational Numbers

Prelude

- **You studied in the primary stage some sets of numbers as :**
  - Set of **COUNTING** numbers $= \{1 , 2 , 3 , 4 , ...\}$
  - Set of **NATURAL** numbers $\mathbb{N} = \{0 , 1 , 2 , 3 , 4 , ...\}$
  - Set of **INTEGERS** $\mathbb{Z} = \{... , -3 , -2 , -1 , 0 , 1 , 2 , 3 , ...\}$
- In this unit , you will recognize another set of numbers called

"**The set of rational numbers**" and it is denoted by the symbol " $\mathbb{Q}$ "

Rational numbers

The numbers : $\frac{1}{2}$, $-\frac{5}{8}$, 3 , 0 , $3\frac{1}{2}$, 0.7 , 2.5 and 15% are **rational numbers**.

WATCH VIDEO

Definition of the rational number

A rational number is a number that can be expressed in the form of a quotient of an integer divided by an integer other than 0

i.e. The rational numbers are all numbers can be expressed as $\frac{a}{b}$
where a and b are integers , $b \neq 0$
, where a and b are called the two terms of the rational number $\frac{a}{b}$

So , **we can express the set of rational numbers as the following :**

The set of rational numbers $\mathbb{Q} = \{x : x = \frac{a}{b} , a \in \mathbb{Z} , b \in \mathbb{Z} , b \neq 0\}$

Based on the previous definition , we can say that :

1 All the decimal numbers are rational numbers.

because any decimal number or decimal fraction can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Examples :

- 2.5 is a rational number can be expressed in the form $\frac{25}{10}$ or $\frac{250}{100}$ or...
- 0.7 is a rational number can be expressed in the form $\frac{7}{10}$ or $\frac{70}{100}$ or...

2 All percents are rational numbers.

because any percentage can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

- 15 % is a rational number can be expressed in the form $\frac{15}{100}$ or $\frac{150}{1000}$ or...

3 All integers are rational numbers.

because any integer can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Therefore :

The set of integers is a subset of the set of rational numbers.

i.e. $\mathbb{Z} \subset \mathbb{Q}$ and since $\mathbb{N} \subset \mathbb{Z}$, then $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

and the following figure shows that.

- 3 is a rational number can be expressed in the form $\frac{3}{1}$ or $\frac{6}{2}$ or $\frac{9}{3}$ or ...
- 0 is a rational number can be expressed in the form $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ or ...
- −16 is a rational number can be expressed in the form $-\frac{16}{1}$ or $-\frac{32}{2}$ or $-\frac{48}{3}$ or ...

! Remark

Each integer is a rational number , but not each rational number is an integer.

For example:

- $\frac{12}{6}$ expresses an integer because : 12 is divisible by 6 and the result is 2
- $\frac{25}{4}$ does not express an integer because : 25 is not divisible by 4

Example 1 **Show why each of the following is a rational number :**

1 $3\frac{2}{5}$ | 2 -0.17

3 0.006 | 4 $27\ \%$

Solution

Each of the previous numbers is a rational number because each of them can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$ as follows :

1 $3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$ | 2 $-0.17 = -\frac{17}{100}$

3 $0.006 = \frac{6}{1000}$ | 4 $27\ \% = \frac{27}{100}$

Remark

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

Example 2 **Choose the correct answer from the given ones :**

1 The number $\frac{5}{2x} \in \mathbb{Q}$, if $x \neq$ .....

(a) 5 (b) 2 (c) 0 (d) -2

2 If $\frac{x-1}{x+3}$ is a rational number , then $x \neq$ .....

(a) -3 (b) -1 (c) 1 (d) 3

3 The number $\frac{3x-12}{2x-4}$ is not rational , if $x =$ .....

(a) -2 (b) 0 (c) 2 (d) 4

Solution

1 (c)

The reason : $\frac{5}{2x} \in \mathbb{Q}$ if $2x \neq 0$ i.e. $x \neq 0$

2 (a)

The reason : since $\frac{x-1}{x+3}$ is a rational number

, then $x + 3 \neq 0$ i.e. $x \neq -3$

3 (c)

The reason : $\frac{3x-12}{2x-4} \notin \mathbb{Q}$ if $2x - 4 = 0$

i.e. $2x = 4$ i.e. $x = \frac{4}{2} = 2$

TRY by yourself 1 *Complete the following table :*

The number	$\frac{5}{x-3}$	$\frac{3}{4-x}$	$\frac{7}{8x}$	$\frac{6x}{x}$
Expresses a rational number if $x \neq$	.....	.....	.....	.....

! Remark

If the rational number $\frac{a}{b} = 0$, then $a = 0$

Example 3 **If the rational number $\frac{x-3}{x+3} = 0$, find the value of x**

Solution

Since $\frac{x-3}{x+3} = 0$

, therefore $x - 3 = 0$ i.e. $x = 3$

TRY by yourself 2 *Complete the following table :*

The rational number	$\frac{x-2}{x-1}$	$\frac{6-x}{x-4}$	$\frac{2x}{x+5}$	$\frac{2x-4}{x+3}$
Equals zero if $x =$	.....	.....	.....	.....

Positive and negative rational number

The rational number $\frac{a}{b}$ is

positive

if the product of its terms is positive

i.e. $a \times b > 0$

and a , b have the same sign.

Examples for positive rational numbers :

- $\frac{3}{5}$
- $\frac{-2}{-3}$

equal to zero

if its numerator is zero

i.e. $a = 0$

notice that zero is not positive nor negative.

Examples for rational numbers equal to zero :

- $\frac{0}{4}$
- $\frac{0}{-2}$

negative

if the product of its terms is negative

i.e. $a \times b < 0$

and a , b have different signs.

Examples for negative rational numbers :

- $\frac{-3}{4}$
- $\frac{2}{-7}$
- $-\frac{4}{5}$

i.e. The set of rational numbers $\mathbb{Q} = \mathbb{Q}_+ \cup \{0\} \cup \mathbb{Q}_-$

Where $\mathbb{Q}_+$ is the set of positive rational numbers , $\mathbb{Q}_-$ is the set of negative rational numbers.

Note that : $\mathbb{Q}_+ \cap \mathbb{Q}_- = \varnothing$

TRY by yourself 3 ***Show which of the following numbers is positive , which is negative and which is zero :***

$\frac{3}{4}$, $\frac{-2}{-9}$, $\frac{\text{zero}}{-5}$, $\left|-\frac{1}{2}\right|$, $\frac{-7}{11}$, $(-5)^2$

Different forms of a rational number

The rational number $\frac{a}{b}$ can be written in the form of another rational number $\frac{c}{d}$ equal to it by applying the following property :

Property

The value of the rational number $\frac{a}{b}$ does not change if its two terms are multiplied or divided by an integer ≠ zero.

For example:

- $\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14}$, $\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

  $\frac{3}{7} = \frac{6}{14} = \frac{9}{21}$

  i.e. $\frac{3}{7}$, $\frac{6}{14}$, $\frac{9}{21}$ are different forms which represent **the same number**.

- $\frac{24}{36} = \frac{24 \div 2}{36 \div 2} = \frac{12}{18}$, $\frac{24}{36} = \frac{24 \div 4}{36 \div 4} = \frac{6}{9}$

  $\frac{24}{36} = \frac{12}{18} = \frac{6}{9}$

  i.e. $\frac{24}{36}$, $\frac{12}{18}$, $\frac{6}{9}$ are different forms which represent **the same number**.

TRY by yourself 4 ***Write in three other forms each of the following rational numbers :***

1 $\frac{2}{3}$ 2 $\frac{16}{64}$

Writing a rational number $\frac{a}{b}$ in its simplest form

For any rational number expressed as $\frac{a}{b}$, we say that this rational number is in its simplest form if each of its terms has the smallest possible value.

For example:

- The simplest form of the rational number $\frac{16}{32}$ is $\frac{1}{2}$ and note that : $\frac{16}{32}$ and $\frac{1}{2}$ represent the same rational number.

$$\frac{16}{32} = \frac{1}{2}$$

So, they represent the same rational number.

- The rational number $\frac{3}{14}$ is in its simplest form and can not be simplified to more simple form.

"To put a rational number $\frac{a}{b}$ in its simplest form , divide each of its terms by the highest common factor (H.C.F.) between them."

Example 4 **Put each of the following numbers in its simplest form :**

1 $\frac{8}{12}$ 2 $-\frac{12}{36}$

Solution

1 The (H.C.F.) of 8 and 12 is 4

Dividing the two terms of $\frac{8}{12}$ by 4, we get : $\frac{8}{12} = \frac{2}{3}$

2 The (H.C.F.) of 12 and 36 is 12

Dividing the two terms of $-\frac{12}{36}$ by 12, we get : $-\frac{12}{36} = -\frac{1}{3}$

TRY by yourself 5 ***Complete the following table :***

The number	$\frac{5}{25}$	$-\frac{6}{9}$	$\frac{27}{45}$	$-\frac{12}{30}$
Its simplest form	.....	.....	.....	.....

Writing the rational number in the form of percentage

To write the rational number in the form of percentage we express it as $\frac{a}{100}$ which means a %

Example 5 **Write each of the following numbers in the form of percentage :**

1 $\frac{9}{20}$ 2 $\frac{5}{16}$ 3 $\frac{17}{1000}$

4 $5\frac{12}{125}$ 5 3.2

Solution

1 $\frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 45\ \%$

Another solution : $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45\ \%$

2 $\frac{5}{16} = \frac{\frac{5}{16} \times 100}{100} = \frac{31.25}{100} = 31.25\ \%$

3 $\frac{17}{1000} = \frac{\frac{17}{1000} \times 100}{100} = \frac{1.7}{100} = 1.7\ \%$

4 $5\frac{12}{125} = \frac{637}{125} = \frac{\frac{637}{125} \times 100}{100} = \frac{509.6}{100} = 509.6\ \%$

5 $3.2 = \frac{32}{10} = \frac{32 \times 10}{10 \times 10} = \frac{320}{100} = 320\ \%$

TRY by yourself 6

Write each of the following numbers in the form of percentage :

1 $\frac{4}{5}$ 2 $\frac{3}{1000}$ 3 2.5

Changing a rational number from the form $\frac{a}{b}$ to a decimal form

Some rational numbers could be changed from the form $\frac{a}{b}$ into a terminating decimal.

For example: • The rational number $\frac{3}{5}$ can be changed into 0.6

• The rational number $\frac{3}{2}$ can be changed into 1.5

$\frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$

$\frac{3 \times 5}{2 \times 5} = \frac{15}{10} = 1.5$

"To write a rational number in the form of a terminating decimal , make its denominator equal to 10 , 100 , 1000 or ..."

Example 6

Write each of the following numbers in the form of a terminating decimal :

1 $\frac{2}{5}$ 2 $|-\frac{3}{8}|$ 3 $-2\frac{7}{25}$

Solution

1 $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

2 $|-\frac{3}{8}| = \frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$

3 $-2\frac{7}{25} = -2\frac{7 \times 4}{25 \times 4} = -2\frac{28}{100} = -2.28$

Check your answer using calculator

TRY by yourself 7 ***Write each of the following rational numbers in the form of a terminating decimal :***

1 $\frac{3}{4}$ 2 $\frac{11}{20}$

! Remark

Some rational numbers could not be changed into terminating decimal as the rational number $\frac{1}{3}$, then using calculator , you find that : $\frac{1}{3} = 0.333333 \ldots$

We express that as $(0.\dot{3})$ and read it as the infinite repeating decimal 0.3 (the recurring decimal 0.3) where the dot above the digit 3 means the digit is repeating (recurring).

Example 7 **Using a calculator , write each of the following rational numbers in the form of a recurring decimal :**

1 $\frac{2}{3}$ 2 $\frac{2}{11}$ 3 $5\frac{71}{333}$

Solution

1 Using the calculator , we get that :

$\frac{2}{3} = 0.6666666667$ i.e. $\frac{2}{3} = 0.\dot{6}$

2 Using the calculator , we get that :

$\frac{2}{11} = 0.1818181818$ i.e. $\frac{2}{11} = 0.\dot{1}\dot{8}$

3 Using the calculator , we get that :

$\frac{71}{333} = 0.2132132132$ i.e. $5\frac{71}{333} = 5.\dot{2}1\dot{3}$

Notice that :

Putting dots above the first and last digits means repeating all digits (first , last and between them)

$5.\dot{2}1\dot{3}$

TRY by yourself 8 ***Write each of the following rational numbers in the form of a recurring decimal :***

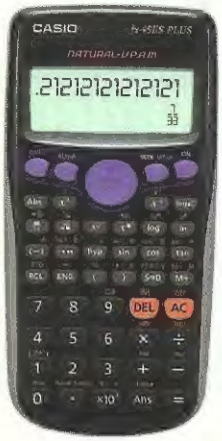
1 $\frac{3}{11}$ 2 $\frac{41}{333}$

Remark

It is possible to write the recurring decimal in the form of $\frac{a}{b}$ by using scientific calculators of type CASIO fX-95ES plus or a different type.

Notice that some scientific calculators can not be able to solve this problem.

For example:



- To write the number $0.\dot{2}\dot{1}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.21212121212121 , then press = you will get the rational number $\frac{7}{33}$
- To write the number $0.13\dot{6}\dot{3}\dot{6}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.1363636363636 , then press = you will get the rational number $\frac{3}{22}$

TRY by yourself 9

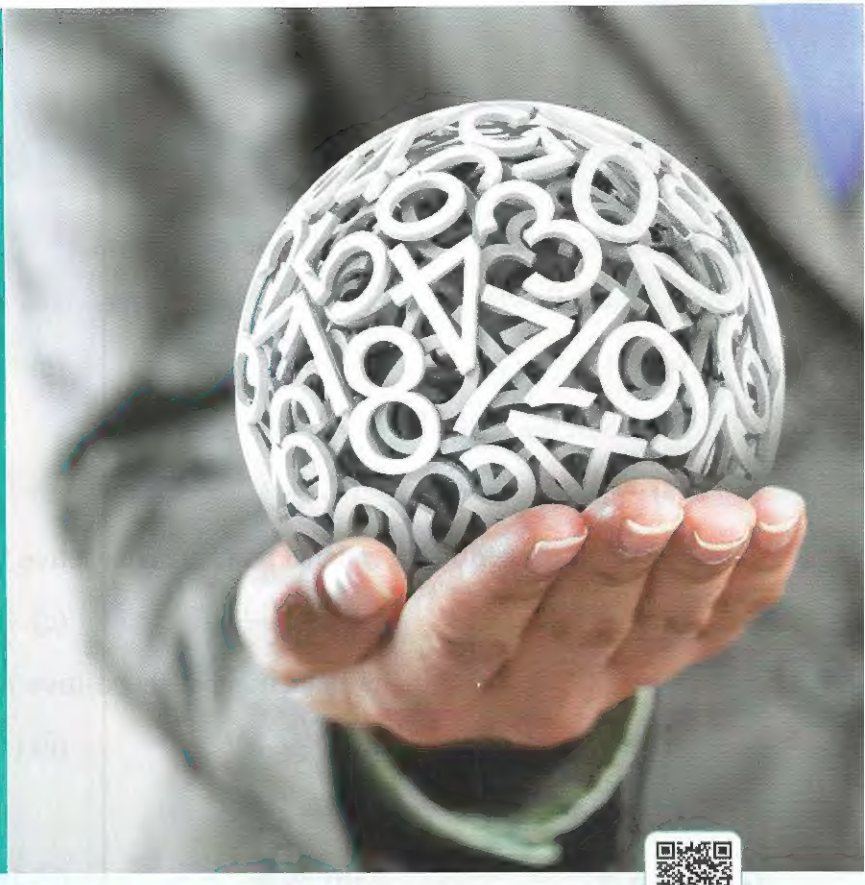
Use the calculator to write each of the following in the form $\frac{a}{b}$ **:**

1 $0.1\dot{5}$

2 $0.14\dot{5}$

Exercise **1**

Set of Rational Numbers



From the school book

● Remember ● Understand ○ Apply Problem Solving

Interactive test

1 **Complete the following :**

1. If $\frac{5}{a}$ is a rational number , then a $\neq$ .....
2. The necessary condition to be $\frac{3}{x-2}$ is a rational number is $x \neq$ .....
3. The number $\frac{2}{3x} \in \mathbb{Q}$ if $x \neq$ .....
4. The number $\frac{x-3}{3x+6}$ is a rational number if $x \neq$ .....
5. The number $\frac{a-6}{a-4}$ is not rational if a = ...
6. The rational number $\frac{x-5}{x} = 0$ if $x =$ .....
7. The rational number $\frac{4-x}{x-3} = 0$ if $x =$ .....
8. The rational number $\frac{5x+15}{x-5} = 0$ if $x =$ .....
9. If $\frac{x+4}{x-3}$ is not rational , then $x - 2 =$ .....
10. $\frac{3}{4} = \frac{9}{.....}$
11. $-\frac{16}{20} = \frac{.....}{10}$
12. $\frac{7}{20} =$ ..... %
13. $\frac{21}{1000} =$ ..... %
14. $|-0.4| =$ ..... %

Remember Understand Apply Problem Solving

2 Choose the correct answer from the given ones :

1 All the following numbers are rational except .....

(a) 0 (b) $\frac{2}{5}$ (c) $\frac{3-3}{7}$ (d) $\frac{4}{5-5}$

2 Which of the following numbers is an integer ?

(a) $-\frac{24}{5}$ (b) $\frac{4}{8}$ (c) $\frac{15}{5}$ (d) $3\frac{1}{4}$

3 Which of the following rational numbers is negative ?

(a) $\frac{0}{-3}$ (b) $-\left|-\frac{1}{2}\right|$ (c) $\frac{-3}{-4}$ (d) $(-7)^2$

4 Which of the following rational numbers is positive ?

(a) $-\frac{3}{4}$ (b) $\frac{0}{5}$ (c) $(-5)^3$ (d) $\frac{-2}{-9}$

5 Which of the following equals $\frac{4}{5}$?

(a) 4 % (b) 54 % (c) 120 % (d) 80 %

6 If $-\frac{4}{5} = \frac{20}{x}$, then $x =$ .....

(a) 25 (b) – 25 (c) 5 (d) 100

7 The rational number $\frac{a}{b}$ is positive if .....

(a) $a\,b > 0$ (b) $a\,b < 0$ (c) $a + b = 0$ (d) $a > b$

8 The rational number $\frac{-7}{a}$ is positive if a ..... zero

(a) > (b) ≥ (c) < (d) =

9 The rational number $\frac{x}{-5}$ is negative if x ..... zero

(a) > (b) < (c) ≤ (d) =

10 If $a = 2$, $b = 6$, then which of the following is not a rational number ?

(a) $\frac{b}{a}$ (b) $-\frac{2}{a}$ (c) $\frac{0}{a+b}$ (d) $\frac{2\,b}{a-2}$

11 $0.\dot{5}\dot{7} =$ .....

(a) $\frac{57}{100}$ (b) $\frac{75}{99}$ (c) $\frac{575}{1000}$ (d) $\frac{19}{33}$

12 $\left|-\frac{8}{25}\right| =$ .....

(a) $-\frac{8}{25}$ (b) $-0.3\dot{2}$ (c) $0.\dot{3}\dot{2}$ (d) 32%

13 12% = ...

(a) $0.\dot{3}$ (b) 1.2 (c) $\frac{3}{25}$ (d) 0.012

3 **Put each of the following numbers in the simplest form :**

1 $\frac{15}{25}$ 2 $-\frac{24}{56}$ 3 $\frac{45}{20}$ 4 $-\frac{132}{88}$

4 **Which of the following rational numbers can be written as a terminating decimal ?**

1 $\frac{7}{15}$ 2 $\frac{7}{20}$ 3 $\frac{5}{8}$ 4 $-\frac{8}{9}$ 5 $\frac{5}{11}$

6 $-\frac{13}{22}$ 7 $\frac{17}{6}$ 8 $2\frac{2}{5}$ 9 $-1\frac{2}{3}$ 10 $\left|-1\frac{2}{9}\right|$

5 **Write each of the following two numbers in the form of a recurring decimal :**

1 $\frac{6}{11}$ 2 $-3\frac{1}{15}$

6 **Write each rational number in the form $\frac{a}{b}$:**

1 -5 2 zero 3 0.75 4 -0.01

5 5.4 6 30% 7 4.5% 8 $8\frac{2}{3}$

7 **Write each of the following rational numbers as a decimal and a percentage :**

1 $2\frac{1}{2}$ 2 $-\frac{3}{20}$ 3 $7\frac{3}{16}$ 4 $\frac{1}{6}$

8 Why does the definition of a rational number $\frac{a}{b}$ state that $b \neq 0$?

For excellent pupils

9 **Choose the correct answer from the given ones :**

1 If $\frac{a}{b}$ is a rational number and ab = zero, then .....

(a) $a = 0$, $b \neq 0$ (b) $a \neq 0$, $b \neq 0$ (c) $a = 0$, $b = 0$ (d) $a \neq 0$, $b = 0$

2 The number $\frac{5x}{|x|-2} \notin \mathbb{Q}$ if x = ...

(a) zero (b) -1 (c) ± 2 (d) 5

10 Write the rational number $\frac{a}{b}$ that equals $\frac{3}{5}$ and the sum of its two terms is 24

11 **If $x \in \mathbb{N}$, find the values of x which make each of the following an integer :**

1 $\frac{75}{x}$ 2 $\frac{15}{x+1}$